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COHERENCE WITHOUT ADDITIVITY

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Coherence without Additivity[1]

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The Dutch book argument is a coherence condition for the existence of subjective probabilities. This note gives a general framework of analysis for this argument in a nonadditive probability setting. Particular cases are given by comonotonic and affinely related Dutch books that lead to Choquet expectations and Min expectations.

Key Words: Coherence, Dutch Book, Constant Linearity, Choquet Expectation, Multiple Priors
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1. INTRODUCTION

The Dutch book argument of de Finetti (1931) is a classic coherence condition for the existence and uniqueness of subjective probabilities. It also provides a standard justification for a model of choice based on subjective expectations. Its appeal and beauty rely on a concept of probability based on everyday life considerations like betting. De Finetti (1976) presented the Dutch book argument as follows.

> "...In English, a combination of bets devised in such a way that, profiting by an inconsistency in the odds given by the bookmaker, someone is certain to win *whatever happens* is called 'Dutch Book' (I don't know why). However, if one wants to, this term could be used to express the condition of consistency that is the sole basis on which the whole theory of probability rests: suffice it to say that it consists in *allowing no chance of a Dutch Book occurring....*"

For a formalization of the meaning of Dutch book, see Wakker (1989). According to Kyburg and Smokler (1964, page 11): "The restriction to coherence thus

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formulates a natural criterion of rationality in situations of uncertainty. Rationality is used in a normative sense here; coherence formulates a criterion of how a person's degrees of belief ought to be related." The debate on the normative aspect of the argument is still going on.

In its original formulation, the argument is not immune to descriptive violations: its behavioral bite has been challenged by experimental evidence (see, e.g., Ellsberg 1961 and Kahneman and Tversky 1979). Consider the following example.

Suppose Bruna decides to purchase an insurance contract for her country house, clearly the insurance payments depend on some states of the world (for example fire, flood, earthquake). Her preferences among contracts are the following: $(3,3,3) \succcurlyeq (12,0,0)$, $(3,3,3) \succcurlyeq (0,12,0)$, and $(3,3,3) \succcurlyeq (0,0,12)$. The contract $(12,0,0)$ means 12 thousands of euros if fire, 0 if flood, and 0 if earthquake. The other contracts are defined similarly. These preferences are behaviorally plausible: in order to obtain a general purpose coverage, Bruna prefers to receive an equal and relatively small reimbursement in all states of the world, than taking the risk of full reimbursement in one state and nothing in the others. Considering all the preferences together yields $(9,9,9) < (12,12,12)$: a Dutch book. It looks like an undesirable result: a set of good decisions, when taken together should still be good.

This note, while using Dutch book arguments, shows how to extend and generalize de Finetti's approach in order to accommodate this kind of descriptive violations. The way to accommodate them is a weaker concept of coherence in a nonadditive environment. The first step is to allow Dutch books only when the involved gambles are not comonotonic (as in Bruna's country house example), thus extending to the infinite case Theorem 6 in Diecidue and Wakker (2001) and providing a general, Dutch book based, characterization of the most popular rank-dependent model of Choquet expected utility (Schmeidler 1989). From an empirical point of view rank-dependent models have received a good deal of attention (see Birnbaum and McIntosh 1996, Bleichrodt and Pinto 2000, Gonzalez and Wu 1999, Harless and Camerer 1994, Tversky and Fox 1995). In particular many works focus on the evaluation of the rank-dependent probabilities (see Abdellaoui 2000, Bleichrodt, van Rijn, and Johannesson 1999, Luce 2000, Tversky and Kahneman 1992).

On the other hand, in some situations, also the assumption of no Dutch books when the bets are comonotonic might be "reasonably" violated. Consider the following example.

John needs a new bike. He has found a second hand one for 40 euros. He is now at the horse races trying to get this amount of money. He is evaluating alternative monetary gambles on a three-horses race and has the following preferences: $(40,50,60) \succcurlyeq (30,80,90)$ and $(40,80,90) \succcurlyeq (60,60,70)$. The gamble $(40,50,60)$ means that John will get 40 euros if the first horse wins the race, 50 if the second wins, and 60 if the third does. The other gambles are defined similarly. The first preference is motivated by the need of the bike: the second gamble involves the risk of not affording it. This time, the gambles are comonotonic, but considering all the preferences together gives $(80,130,150) < (90,140,160)$: a Dutch book.

This calls for an additional generalization, which is the main contribution of this note. Allowing Dutch books only when the involved gambles are not affinely related, we provide a Dutch-book-based characterization of invariant biseparable preferences (see Ghirardato and Marinacci 2001, and Ghirardato, Maccheroni, and Marinacci 2001). Moreover, with a further uncertainty aversion assumption, we get Min expected utility (Gilboa and Schmeidler 1989) the generalized expected utility model most successful for finance applications (see Epstein and Wang 1994 and

Epstein and Zin 1989).

Finally, considering the minimal requirement of no Dutch books existing when the involved gambles are sure prospects, we obtain a very general, still appealing, concept of prevision.

To sum up. De Finetti, via the Dutch book argument, justified a model of choice based on expectations. We extend his approach in such a manner that the generalized argument can provide a new foundation for some very general nonexpected utility models of choice. These models are more and more popular and successful in economics and psychology, see for example Camerer (1999) and Starmer (2000). From an applied point of view, QALY evaluations of health policies in a rank-dependent spirit have received increasing attention (Bleichrodt, van Rijn, and Johannesson 1999, Miyamoto 1988, 1999).

In the next section we present the result, then conclusions follow.

2. GENERALIZED DUTCH BOOKS AND COHERENCE

2.1. The set-up

We consider a standard subjective setting; bets are simply "...wagers on any facts and for any amount..." (de Finetti 1976). Formally, let S be the set of *states of the world* and Σ be the *events* algebra. A *bet* is represented by a simple measurable function $f = \sum_{i=1}^{N} \alpha_i \mathbf{1}_{A_i}$ (the bettor wins α_i euros if event A_i obtains), or, in general, by a bounded measurable function f (the bettor wins $f(s)$ euros if state s obtains). The set of all simple bets is $B_0 = B_0(S, \Sigma)$, while the set of all bets is $B = B(S, \Sigma)$: the supnorm closure of B_0. As usual we identify the real number α with the *constant* bet $\alpha \mathbf{1}_S$.

Two bets f and g are *comonotonic* if $[f(s) - f(s')][g(s) - g(s')] \geq 0$ for all $s, s' \in S$ (see Schmeidler 1986 and 1989); while they are *affinely related* if there exists $\alpha \geq 0$ and $\beta \in \mathbb{R}$ such that $f = \alpha g + \beta$ or $g = \alpha f + \beta$ (see Gilboa and Schmeidler 1989 and Ghirardato, Klibanoff, and Marinacci 1998). These concepts are now commonly used and their meanings well discussed in the literature, so we will not indulge on them here.

Let $\succcurlyeq$ be a binary relation on B representing the preferences of a decision maker among the bets; as usual, $\succ$ denotes the asymmetric part of $\succcurlyeq$, and $\sim$ denotes the symmetric one. The crucial notion is the following.

Definition 1. A *Dutch book* consists of two arrays of simple bets $f^1, ..., f^M$, $g^1, ..., g^M \in B_0$ such that $f^j \succcurlyeq g^j$, for all $j = 1, ...M$, but $\sum_{j=1}^{M} f^j(s) < \sum_{j=1}^{M} g^j(s)$, for all states $s \in S$.

This notion represents something undesirable, therefore *coherence* requires that no Dutch book is allowed. Whenever all the involved bets are pairwise comonotonic we call the book a *comonotonic Dutch book*, while if they are affinely related we call it *affine Dutch book*, finally when they are constant we call it *trivial Dutch book*.

In the sequel we make use of the following properties of $\succcurlyeq$.

- Weak Order: For all f and g in B: $f \succcurlyeq g$ or $g \succcurlyeq f$. For all f, g, and h in B: if $f \succcurlyeq g$ and $g \succcurlyeq h$, then $f \succcurlyeq h$.

- Monotonicity: For all f and g in B: if $f(s) \geq g(s)$ for all $s \in S$, then $f \succcurlyeq g$.

- Fair Price: For each f in B, there exists $\xi = \xi(f) \in \mathbb{R}$ such that $f \sim \xi$.[3]
- Uncertainty Aversion: For all f and g in B and all α in $(0,1)$: $f \sim g$ implies $\alpha f + (1-\alpha) g \succcurlyeq f$.

Before stating the result a few more ingredients are needed. A functional $V : B \to \mathbb{R}$ is *monotonic* if $V(f) \geq V(g)$ whenever $f \geq g$, it is a *prevision* if it is monotonic and $V(\beta) = \beta$ for all $\beta \in \mathbb{R}$, finally it is *constant-linear* if $V(\alpha f + \beta) = \alpha V(f) + \beta$ for all $f \in B$, $\alpha \geq 0$ and $\beta \in \mathbb{R}$.

2.2. The result

We can now state the anticipated representation result.

Theorem 1. *A binary relation $\succcurlyeq$ on B is a monotonic weak order that satisfies the fair price property and allows no trivial Dutch books iff there exists a prevision $V : B \to \mathbb{R}$ representing $\succcurlyeq$. Moreover:*

(i) the relation $\succcurlyeq$ allows no affine Dutch books iff the prevision V is constant-linear;

(ii) the relation $\succcurlyeq$ allows no comonotonic Dutch books iff there exists a capacity C such that $V(f) = \int_S f dC$;

(iii) the relation $\succcurlyeq$ allows no Dutch books iff there exists a probability P such that $V(f) = \int_S f dP$.

The prevision V is unique.

As anticipated in the Introduction, (i) characterizes invariant biseparable preferences, (ii) - extending to the infinite case Diecidue and Wakker (2001) - yields Choquet expectations, while (iii) is the famous de Finetti Dutch book Theorem. Among the many works referring to point three we mention: Berti, Regazzini, and Rigo (2002), Berti and Rigo (2000), Blackwell and Girshick (1954), Cifarelli and Regazzini (1996), Coletti (1990), Crisma, Gigante, and Millossovich (1997), Holzer (1985), Regazzini (1985), and Wakker (1989).

Proof. If $f \sim \xi$, set $V(f) = \xi$. V is well defined, in fact the fair price is unique. Assume the contrary: there exist $f \in B$ and $\xi > \gamma \in \mathbb{R}$ such that $f \sim \xi$ and $f \sim \gamma$, hence $\gamma \succcurlyeq \xi$, but $\gamma < \xi$: a trivial Dutch book. It is easy to verify that V represents $\succcurlyeq$, in particular V is monotonic.

Assume $\succcurlyeq$ allows no affine Dutch books. Next we show that $V(f+\beta) = V(f) + \beta$ for all $f \in B_0$ and all $\beta \in \mathbb{R}$. Let $V(f+\beta) = \gamma$, $V(f) = \xi$ and, by contradiction $\gamma > \xi + \beta$. The bets $f+\beta, \xi, f$, and γ are affinely related,[4] and $f + \beta \succcurlyeq \gamma$, $\xi \succcurlyeq f$, but $f + \xi + \beta < \gamma + f$: an affine Dutch book.

In the same manner: $V(nf) = nV(f)$ for all $f \in B_0$ and all $n \in \mathbb{N}$. Assume $V(nf) = \gamma$, $V(f) = \xi$ and $\gamma > n\xi$. The bets nf, ξ, f, and γ are affinely related, and $nf \succcurlyeq \gamma$, $\xi \succcurlyeq f$, but $nf + n\xi < nf + \gamma$: an affine Dutch book. Hence, $V(qf) = qV(f)$ for all $f \in B_0$ and all $q \in \mathbb{Q}_+$.

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[3] $\xi(f)$ is also called *certainty equivalent* of f.

[4] Notice that for all $f \in B$ and all $\xi \in \mathbb{R}$, $\xi = 0f + \xi$ implies that f and ξ are affinely related.

Let $f_n \in B_0$, and assume $f_n \to f \in B$ uniformly. There exists $\{\gamma_n\} \subseteq \mathbb{R}_+$ such that $\gamma_n \to 0$ and $f_n - \gamma_n \leq f \leq f_n + \gamma_n$, hence

$$V(f_n) - \gamma_n = V(f_n - \gamma_n) \leq V(f) \leq V(f_n + \gamma_n) = V(f_n) + \gamma_n,$$

therefore $V(f_n) \to V(f)$. In particular, V is supnorm continuous in B_0.

Let $\alpha > 0$, $f \in B_0$ and $\{q_n\} \subseteq \mathbb{Q}_+$ such that $q_n \to \alpha$. Hence $q_n f \to \alpha f$ uniformly, so $V(\alpha f) = \lim_n V(q_n f) = \lim_n q_n V(f) = \alpha V(f)$. In sum, V is constant-linear and supnorm continuous on B_0. By uniform approximation with simple measurable functions, it is easy to show that V is constant-linear and continuous on the whole B.

Next we show that a preference represented by a constant linear prevision allows no affine Dutch books. By contradiction, let $f^1 \succcurlyeq g^1, ..., f^M \succcurlyeq g^M$ in B_0 be all affinely related, and $\sum_{j=1}^{M} f^j(s) < \sum_{j=1}^{M} g^j(s)$ for all states s. Then $V(f^1) \geq V(g^1), ..., V(f^M) \geq V(g^M)$, and by constant linearity $V\left(\sum_{j=1}^{m} f^j\right) \geq V\left(\sum_{j=1}^{M} g^j\right)$. But $\sum_{j=1}^{M} f^j(s) < \sum_{j=1}^{M} g^j(s)$, for all states s, and V, being monotonic and constant linear, is strictly monotonic on B_0, so we have $V\left(\sum_{j=1}^{M} f^j\right) < V\left(\sum_{j=1}^{M} g^j\right)$, which is absurd. This concludes the proof of (i).

Assume $\succcurlyeq$ allows no comonotonic Dutch books. Next we show that $V(f+g) = V(f) + V(g)$ for all comonotonic $f, g \in B_0$. Let $V(f+g) = \gamma$, $V(f) = \xi$ and $V(g) = \theta$ and $\gamma > \xi + \theta$. Then $f+g, \xi, \theta, \gamma, f, g$ are comonotonic,[5] $f+g \succcurlyeq \gamma$, $\xi \succcurlyeq f$, $\theta \succcurlyeq g$, but $f+g+\xi+\theta < \gamma+f+g$: a Comonotonic Dutch book. By Schmeidler (1986) there exists a capacity C such that $V_{|B_0}(f) = \int f dC$ for all $f \in B_0$. V and $\int \cdot dC$ are both monotonic and constant linear, hence continuous in B, and they coincide on B_0 which is dense in B, then $V = \int \cdot dC$. Which proves (ii), since the same argument we used to exclude affine Dutch books for preferences represented by a constant linear prevision now permits to exclude comonotonic Dutch books for preferences represented by a Choquet integral. Clearly, the well known (iii) can be obtained in a similar way. ∎

Next we consider the consequences of uncertainty aversion on constant-linear previsions.

Corollary 1. *Let $\succcurlyeq$ be a relation on B represented by a constant linear prevision $V : B \to \mathbb{R}$. The relation $\succcurlyeq$ is uncertainty averse iff there exists a (unique) compact and convex set $\mathcal{C}$ of probability charges such that $V(f) = \min_{P \in \mathcal{C}} \int_S f dP$.*

Notice that, in the special case in which $V(f) = \int f dC$ this simply means that $\succcurlyeq$ is uncertainty averse iff C is a convex capacity. While the case in which $V(f) = \int f dP$ is characterized by uncertainty neutrality.

Proof. Similar to Lemma 3.3 in Gilboa and Schmeidler (1989). ∎

[5] Just remember f and g are comonotonic, and apply the definition of comonotonicity.

3. CONCLUSIONS

The Dutch book argument is a now classical coherence condition for the existence of subjective probabilities. In this note we have investigated the possibility of using this type of argument to shed more light on some successful nonexpected utility models. The result is formally unifying and is still suggestive, even if it may lack some normative contents (which, however face some criticism also in the original formulation, see e.g. Schick 1986). On the other hand, it is consistent with a descriptive approach. As a matter of fact, it is possible to construct examples showing that Dutch books can pop up in real life situations.

REFERENCES

Abdellaoui, M. (2000), Parameter-Free Elicitation of Utilities and Probability Weighting Functions, Management Science 46, 1497–1512.

Berti, P., E. Regazzini, and P. Rigo (2002), Coherent Prevision of Random Elements, Journal of the Italian Statistical Society, forthcoming.

Berti, P. and P. Rigo (2000), Integral representation of linear functionals on spaces of unbounded functions, Proceedings of the American Mathematical Society 128, 3251-3258.

Blackwell, D. and M.A. Girshick (1954), "Theory of Games and Statistical Decisions." Wiley, New York.

Birnbaum, M. H. and W. R. McIntosh (1996), Violations of Branch Independence in Choices between Gambles, Organizational Behavior and Human Decision Processes 67, 91–110.

Bleichrodt, H., J. van Rijn, and M. Johannesson (1999), Probability Weighting and Utility Curvature in QALY-Based Decision Making, Journal of Mathematical Psychology 43, 238-260.

Bleichrodt, H. and Jose Luis Pinto (2000), A Parameter-Free Elicitation of the Probability Weighting Function in Medical Decision Making, Management Science 46, 1485-1496.

Camerer, C. (1999), Prospect Theory in the Wild: evidence from the field, in D. Kahneman and A. Tversky, eds., Choices, Values and Frames, forthcoming.

Cifarelli, D.M. and E. Regazzini (1996), de Finetti's Contribution to Probability and Statistics, Statistical Science 11, 253-282.

Coletti, G. (1990), Coherent Qualitative Probabilities, Journal of Mathematical Psychology 34, 297-310.

Crisma, L., P. Gigante, and P. Millossovich (1997), A Notion of Coherent Prevision for Arbitrary Random Quantities, Journal of the Italian Statistical Society 6, 233-243.

de Finetti, B. (1931), Sul Significato Soggettivo della Probabilità, Fundamenta Mathematicae 17, 298-329. Translated into English as: On the Subjective Meaning of Probability, in P. Monari and D. Cocchi, 1993, eds., Probabilità e Induzione, Clueb, Bologna, 291-321.

de Finetti, B. (1976), La Probabilità: Guardarsi dalle Contraffazioni!, Scientia 111, 255-281. Translated into English by I. McGilvray: Probability: Beware of Falsifications!, in H. E. Kyburg Jr. and H. E. Smokler, Studies in Subjective Probability; 2nd edition 1980, 193-224, Krieger, New York.

Diecidue, E. and P. P. Wakker (2001), Dutch Books: Avoiding Strategic and Dynamic Complications, and a Comonotonic Extension, Mathematical Social Sciences, forthcoming.

Ellsberg, D. (1961), Risk, Ambiguity and the Savage Axioms, Quarterly Journal of Economics 75, 643–669.

Ghirardato, P., P. Klibanoff, and M. Marinacci, (1998), Additivity with Multiple Priors, Journal of Mathematical Economics 30, 405-420.

Ghirardato, P. and M. Marinacci (2001), Risk, Ambiguity, and the Separation of Utility and Beliefs, Mathematics of Operations Research 26, 864-890.

Ghirardato, P., F. Maccheroni, and M. Marinacci (2001), A Fully Subjective Perspective on Ambiguity, Mimeo.

Gilboa, I. and D. Schmeidler, (1989), Maxmin expected utility with non-unique prior, Journal of Mathematical Economics 18, 141-153.

Gonzalez, R. and G. Wu (1999), On the Shape of the Probability Weighting Function, Cognitive Psychology 38, 129–166.

Harless, D. W. and C. Camerer (1994), The Predictive Utility of Generalized Expected Utility Theories, Econometrica 62, 1251–1289.

Holzer, S. (1985), On Coherence and Conditional Prevision, Unione Matematica Italiana, Bollettino, C, Serie VI, Analisi Funzionale e Applicazioni, 4-C (1), 441-460.

Kahneman, D. and A. Tversky (1979), Prospect Theory: An Analysis of decision under risk, Econometrica 47, 263-291.

Kyburg, H. E. Jr. and H. E. Smokler, (1964), eds., Studies in Subjective Probabilities, Wiley, New York, 2nd edition 1980, Krieger, New York.

Luce, D. (2000), Utility of Gains and Losses: Measurement-Theoretical and Experimental Approaches. Lawrence Erlbaum Publishers, London.

Miyamoto, J. M. (1988), Generic Utility Theory: Measurement Foundations and Applications in Multi-Attribute Utility Theory, Journal of Mathematical Psychology 35, 357-404.

Miyamoto, J. M. (1999), Quality-Adjusted Life Years (QALY) Utility Models under Expected Utility and Rank Dependent Utility Assumptions, Journal of Mathematical Psychology 43, 201-237.

Regazzini, E. (1985), Finitely Additive Conditional Probabilities, Rendiconti del Seminario Matematico e Fisico di Milano 55, 69-89.

Schick, F. (1986), Dutch Bookies and Money Pumps, Journal of Philosophy 83, 112-119.

Schmeidler, D. (1986), Integral Representation Without Additivity, Proceedings of the American Mathematical Society 97, 253-261.

Schmeidler, D. (1989), Subjective Probability and Expected Utility without Additivity, Econometrica 57, 571–587.

Starmer, C. (2000), Developments in Non-Expected Utility Theory: The Hunt for a Descriptive Theory of Choice under Risk, Journal of Economic Literature, forthcoming.

Tversky, A. and C. R. Fox (1995), Weighing Risk and Uncertainty, Psychological Review 102, 269–283.

Tversky, A. and D. Kahneman (1992), Advances in Prospect Theory: Cumulative Representation of Uncertainty, Journal of Risk and Uncertainty 5, 297–323.

Wakker, P. P. (1989), Additive Representations of Preferences, A New Foundation of Decision Analysis. Kluwer Academic Publishers, Dordrecht.

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